- Determinants represent the signed area or volume of the row or column vectors of an $n \times n$ matrix in n-space.
  - 2-space: Let $\vec{a} = \langle a_1,\ a_2 \rangle$, $\vec{b} = \langle b_1,\ b_2 \rangle$. The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ is $|\vec{a} \times \vec{b}| = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$
  - 3-space: **Triple scalar product**: $\vec{a} \cdot (\vec{b} \times \vec{c})$
    - Let $\vec{a} = \langle a_1,\ a_2,\ a_3 \rangle$, $\vec{b} = \langle b_1,\ b_2,\ b_3 \rangle$, and $\vec{c} = \langle c_1,\ c_2,\ c_3 \rangle$.
    - $\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
    - Represents the volume of the **parallelepiped** formed by $\vec{a}$, $\vec{b}$, and $\vec{c}$.
    - The tetrahedron formed by $\vec{a}$, $\vec{b}$, and $\vec{c}$ has one-sixth of this volume.
  - Determinants are a good scalar representation of vector spaces.
  - This is also intuitive to why if one row or column is a linear combination of other rows or columns, then the determinant is 0.
- **Lines** in three-space:
  - Let $\vec{r}_o$ be the position vector of a point on a line $\vec{r}$ in three-space, and let the direction vector $\vec{L}$ be parallel to $\vec{r}$.
  - Parametric equation: $\vec{r}(t) = \vec{r}_o + t\vec{L}$
  - Symmetric equation: $\dfrac{x - x_o}{a} = \dfrac{y - y_o}{b} = \dfrac{z - z_o}{c}$
    - Note: Three variables and two constraints. One degree of freedom.
- **Planes** in three-space:
  - Let $\vec{r}_o$ be the position vector of a point on a plane in three-space, and let the vectors $\vec{a}$ and $\vec{b}$ lie in the plane.
  - $\vec{n} = \vec{a} \times \vec{b} = \langle a, b, c \rangle$ is normal to the plane
  - Vector equation: $\vec{r}(s,t) = \vec{r}_o + s\vec{a} + t\vec{b}$
  - Parametric equation: $\vec{r}(s,t) = \langle x(s,t),\ y(s,t),\ z(s,t) \rangle$
  - $ax + by + cz = d$, where $\vec{r}_o \cdot \vec{n} = d$ and $\vec{n} = \langle a, b, c \rangle$
    - Note: Three variables and one constraint. Two degrees of freedom.